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METHODOLOGY FOR ANALYSIS AND SYNTHESIS OF CONTROL ALTERNATIVES

METODOLOGIA PARA EL ANALISIS Y SINTESIS DE ALTERNATIVAS DE CONTROL

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Abstract

This work describes a methodology for the assessment of control alternatives of a given process. With this methodology once the process model is developed, it is linearized to evaluate different control criteria aiming to discern between viable combinations of measurements and controls. To cover the different alternatives of control pairs, all the possible alternatives are generated; then, for a rapid reduction of the number of candidates, the most strict control criteria are applied first. Later, less strict control criteria are applied sequentially to the remaining candidates. The most promising candidates are used to synthesize a decoupled control obtained as a precompensator optimized for diagonal dominance. This methodology is implemented using a pattern-based paradigm, which is applied to the analysis of a separation by a double effect evaporator and two distillation process with different degrees of detail.

Keywords: control assessment, screening control alternatives, control decoupling.

Resumen

Este trabajo describe una metodología para el análisis de alternativas de control en un proceso determinado. Con esta metodología, una vez obtenido el modelo matemático, se lineariza automáticamente para evaluar distintos criterios de control a fin de discernir entre combinaciones viables de mediciones y controles. Para evaluar todas las alternativas de control, primero se generan todas las posibles alternativas, y después se aplican los criterios de control más estrictos para reducir rápidamente el número de candidatos. Posteriormente, se aplican secuencialmente los criterios de control deseables a los candidatos restantes.
Las configuraciones más promisorias se usan para sintetizar un control desacoplado mediante un compensador que optimiza la dominancia en la diagonal. Esta metodología fue implementada usando un paradigma basado en patrones, el cual se aplica al análisis de un evaporador doble efecto y a dos columnas de destilación con grados de fidelidad diferentes.

*Palabras clave***:** alternativas de control, evaluación de alternativas de control, control desacoplado.

1. Introduction

High fidelity model is required to analyze a process plant at different operating conditions. A mathematical model of this process is usually specified in terms of balance equations (mass, moment and energy) for a given set of state variables, physical properties, transfer mechanisms (constitutive equations), and constraints (equilibrium, limits of validity of correlations). Once the model is formulated, the steady state is obtained allowing the model to stabilize during a span of simulation time. The feasible measurements are also

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identified; model parameters, such as transfer coefficients, are tuned to fit steady-state response. Other model parameters, such as model capacities, are tuned to fit dynamic response. The mathematical representation of this model can be grouped as:

$h(\mathbf{p}, \mathbf{x}, \mathbf{u}, \mathbf{v}) = 0$	Constitutive equations	(1)
$\mathbf{r}(\mathbf{p}, \mathbf{u}, \mathbf{x}, \mathbf{y}) \leq 0$	Constraints	(2)
$\mathbf{M}(\mathbf{x}) \dfrac{d\mathbf{x}(t)}{dt} = \mathbf{f}(t, \mathbf{p}, \mathbf{u}, \mathbf{v}, \mathbf{x})$	Balance equation	(3)
$\mathbf{y} = g(\mathbf{p}, \mathbf{u}, \mathbf{v}, \mathbf{x})$	Measurements equations	(4)

To determine the control of this type of process there is a difficulty involved in translating a model from a physical representation into a state transfer function that commonly leads to the use of a dual type of modeling: one for dynamic behavior, and another for control analysis. The task of translating a given model increases when we have to produce a transfer function for every alternative of configuration pairings.

The selection of the measurements and controls is a problem of combinatorial type, common to problems of decision-making. In order to reduce the alternatives, one strategy is to subdivide the process as subsystems but usually interaction depends upon operating conditions; another strategy is to use heuristics criteria. Both strategies for control configuration can cause a sub-optimal operation of the process.

In this paper, we present advances in how to deal with the same model for both dynamic behavior and control analysis, and how to select control alternatives. Later, we mention a linearization scheme, and how a set of control criteria are applied to select feasible control pairs of the linearized model. For multivariable control an optimized decoupled is used. This methodology is applied to several examples. Finally, conclusions are derived from the use of this methodology.

2. Methodology for the analysis of control alternatives

To evaluate the feasible control alternatives from a mathematical representation, Luyben (1970) fitted the bode plots to obtain the transfer function of the process; Newell and Fisher (1972) used finite differences; Jacobsen et al (as reported by Skogestad, 1992) obtained a linearized model with subsequent model reduction using the Hankel model; Gross *et al.* (1998) used the ARX model to obtain a linear transfer function. Brosilow and Magaña (2002) used automatic differentiation. In this work the linear model was obtained using the Matlab procedure numjac for a given function h(z,t). This procedure adjusts the perturbation step to reduce rounding error. To analyze control alternatives we used the augmented vector of variables z = [x, u] and the augmented vector of functions l = [fM^{-1}, g].Thus, the linearized model is obtained (eqns. 5,6), by numerical differentiation:

$$\frac{dx}{dt} = Ax(t) + Bu(t) \tag{5}$$

$$y = Cx(t) + Du(t) \tag{6}$$

Then by invoking numjac ('ProcessModel', t, z) the numerical Jacobian can be obtained. The matrices of partial derivatives are given by: $A(t) = \dfrac{\partial f}{\partial x}; B(t) = \dfrac{\partial f}{\partial u}; C(t) = \dfrac{\partial g}{\partial x}; D(t) = \dfrac{\partial g}{\partial u}$

The transfer matrix is:

$$G(s) = C(sI - A)^{-1}B + D = \frac{Num(s)}{Den(s)}$$

The Gain matrix can be decomposed as: $G = U\Sigma V^T$. Where U, V are orthogonal matrices containing the left (or output) and right (or input) singular vector, and Σ is the matrix with singular values.

Deshpande (1989) recommends pairing the vector component of U_1 with the manipulated variable associated with the largest vector component of V_1. Then he suggests pairing the largest vector component U_2 with the largest manipulated vector component of V_2, etc. Finally he recommends that manipulated variables be considered in terms of the range over which they are varied, instead of their absolute value. One of the characteristics of the SVD is that it is scale sensitive. The ratio of singular values indicates how sensible is the process to change on the type of metric system, or to the accuracy of instruments at different frequencies of the process.

2.1 Description of control criteria

The control design deals with: a) The selection of controlled variables, b) The selection of measurements, c) The selection of control configuration, and d) The selection of controller type. In general the number of alternative configurations of *Ny* sets of measurements taken from $N_{Tot}y$, and *Nu* sets of controls taken indistinctly from $N_{Tot}u$ is:

$$\left(\frac{N_{Tot}y!}{(N_{Tot}y - Ny)!Ny!}\right) * \left(\frac{N_{Tot}u!}{(N_{Tot}u - Nu)!Nu!}\right)$$

The requirements of a suitable control configuration are listed in table 1 (Groenedijk *et al.*, 2000; Skogestad, 1999):

Table 1. Control requirements and their mathematical criteria.

Analysis	Evaluation	Criteria
Model	Number of manipulated variables.	the number of Degrees of freedom.
Posing	Number of independent states.	The rank of A(t).
Structural	Number of measured states.	Observability matrix.
	Number of controlled states.	Controllability matrix.
Stability	The oscillatory or unstable condition is determined.	location of poles and zeroes in the Laplace domain (RH-plane).
Sensitivity	Ratio of the maximum and the minimum singular values (Condition number)	Smallest condition numbers
Interaction	Interaction between subprocesses.	Relative Gain Array (RGA).
Pairings	Measurements and controllers	
Synthesis	**Strategy**	
Sensitivity	Scaling	Diagonal dominance
Interaction	Decoupling Interaction.	Column diagonal dominance
Tuning	Performance criteria	Minimization
Feedback-stability	Nyquist array	Encirclements of Direct Nyquist Array.

2.1.1 Model posing

All states must be independent; otherwise, the model needs to be reformulated.

2.1.2 Control interactions

If there is little interaction between controls then the relative gain array $\Lambda = G(s) \otimes G^{-T}(s)$ has elements close to unit in its diagonal. Bristol (see Shinskey, 1977) approximated this way the interaction between an input u_i and an output y_j. This array is evaluated in the rank of frequencies of the system. To complement the analysis, McAvoy (1998) recommends the use of the Relative Gain Array, along with considerations for integrated variables.

2.1.3 Synthesis

Even though a large amount of work is developed currently on model predictive control (see Morari and Lee, 1999), when an operator needs to place a loop in manual mode he must know which input has a dominant effect on a given measurement. A desired precompensator would make $Q(s) = K(s)G(s) \approx I$ to complete diagonalize the process. Thus $K(s) = G(s)^{-1}$, but this approach would lead to high order precompensator. Some simplifications can be performed; Luyben (1970) used an ideal decoupling compensator: $K = G^{-1}Diag(G)$ to cancel the effect of interaction. Other strategies for decoupling are: block triangulation of the system (Waller *et al.*, 2003), and to synthesize a combined compensators for high, intermediate and low frequency (Dutton *et al.*, 1997). Skogestad and Postlethwaite (2001) warn us about the fact that decoupling *K(s)* matrix can add new zeroes to the matrix Q(s) and increase the order of the system. Bryant and Yeung (1996) proposed a methodology based on the diagonal dominance (instead of cancellation of effects) for control synthesis, which includes scaling and a decoupling precompensator. This methodology was applied in this work and it is described in the following paragraphs.

2.2 Scaling

The ratio of singular values indicates the extent in which scaling is necessary. Once the input-output pairing is selected, a scaling is designed to produce diagonal dominance by columns. $|G_{jj}| \geq \sum_{\substack{i=1 \\ i \neq j}}^{n} |G_{ij}|$

This means that some interaction among loops will remain, but the dominant effect will be $y_j = g_{jj}u_j$. Diagonal dominance also allows equating the sum of the encirclements of the diagonal elements as the original encirclements of Nyquist plots of the diagonal elements z_{jj} (Deshpande, 1989). Thus for a given gain matrix G(s), the scaling selected produce diagonal dominance by columns:

$$\min_{w>0} \max_{j} \sum_{\substack{i=1 \\ i \neq j}}^{n} \frac{w_i}{w_j} \left| \frac{g_{ij}(s)}{g_{jj}(s)} \right|, \; j = 1, \ldots, n$$

$$s.t. \sum_{i=1}^{n} w_i = 1$$

To account for the effect of a range of different frequencies, ω, the optimal scaling over all the feasible frequencies Ω is obtained as:

$$\min_{w>0} \max_{\omega \in \Omega} \sum_{\substack{i=1 \\ i \neq j}}^{n} \frac{w_i}{w_j} \left| \frac{g_{ij}(s)}{g_{jj}(s)} \right|, \; j = 1, \ldots, n \tag{7}$$

A minmax optimizer was used to evaluate w_i.

2.3 Parameterized precompensator

Pre-compensation is used to effectively achieve column dominance of a compensated process over a set of frequency range,Ω. Since the ith column dominance measure of a precompensator system Q(s) = G(s) K(s) is independent of other columns of the precompensator K(s), the problem can be decomposed in several subproblems. Each one involves finding a column K(s). The total number of variables of every subproblem is equal to the number of rows of K(s), then the formulation to obtain the compensator K(s) is:

$$\min_{w>0} \max_{\omega\in\Omega} \sum_{\substack{i=1 \\ i\neq j}}^{n} \left| \frac{q_{ij}(s)}{q_{jj}(s)} \right|, j = 1,...,n$$
$$s.t. K_l \leq K_{ij} \leq K_u \tag{8}$$
$$and$$
$$0 \leq \sum_{i=1}^{n} |K_i| \leq K_{\max}$$

Lower and upper bounds were specified as the combination of linear inequalities to preserve the linearity of the programming problem. Every term in the precompensator has a frequency dependent structure

$$K(s) = \sum_{i=-1}^{P} s^i K^i,$$

P=1 for a PID compensator.

Precompensator design was evaluated by a global minimization (Finkel, 2003). This optimizer obtained higher quality results than minmax optimizers due to the non-continuous nature of the problem.

2.4 Synthesis methodology

The dynamic response is used to evaluate the stability and the range of frequencies in the process. The proposed control configuration appears in fig. 1. When the matrix gain is obtained, we execute the following procedure (Deshpande, 1989):
Select an appropriate frequency range: $0 \leq \omega \leq \omega_c$, at higher ω_c the process response remains negligible. Obtain K(s) such that Q(s) = G(s)*K(s) is diagonally dominant. Verify dominance calculating the process Gershgorin bands for every control. The open loop system is diagonal dominant if the Gershgorin circles exclude the origin (0, 0). The closed loop system is diagonal dominant and stable if the Gershgorin circles exclude $(-\frac{1}{f_i}, 0)$, where the gain f_i is the point of intersection of the inner boundary of the Gershgorin band with the negative real axis for open loop stable systems. Design feedback networks for the individual loops. Individual PID controls can be implemented by a lead-lag precompensator $K\frac{(1+\tau_1 s)}{(1+\tau_2 s)}$, which fits a performance criterion $\int e^T w e\, dt$.

2.5 Control patterns

In order to screen conveniently all the possible alternatives, first we tested the *essential* control criteria, thus we automatically diagnose if a given configuration succeeds in this criteria. Later on, we tested the *necessary* criteria; if they are not successful, the configuration is discarded. The remaining configurations are candidates for design. *Preferred* criteria are sequentially applied for the remaining configurations. Once the model is constructed and validated, the analysis and design have a regular sequence. Thus appropriate tools can be developed. During the process of model building, design-patterns allow an agile model construction. Models have the procedures shown in table 2 (cf. Muñoz-Arteaga 2003):

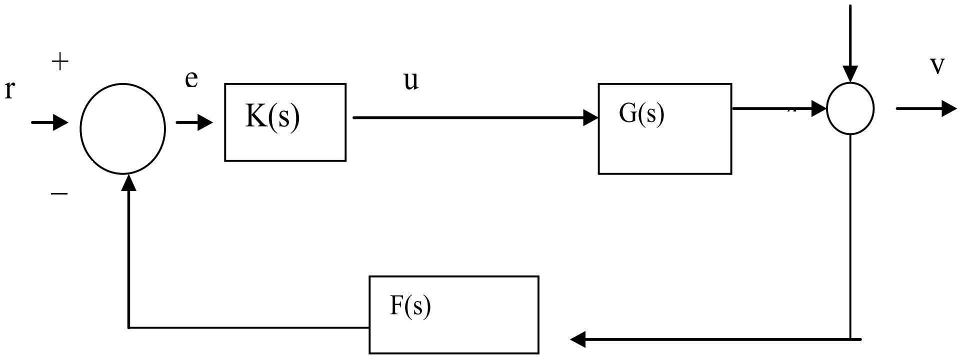

Fig. 1. Feedback system with compensation.

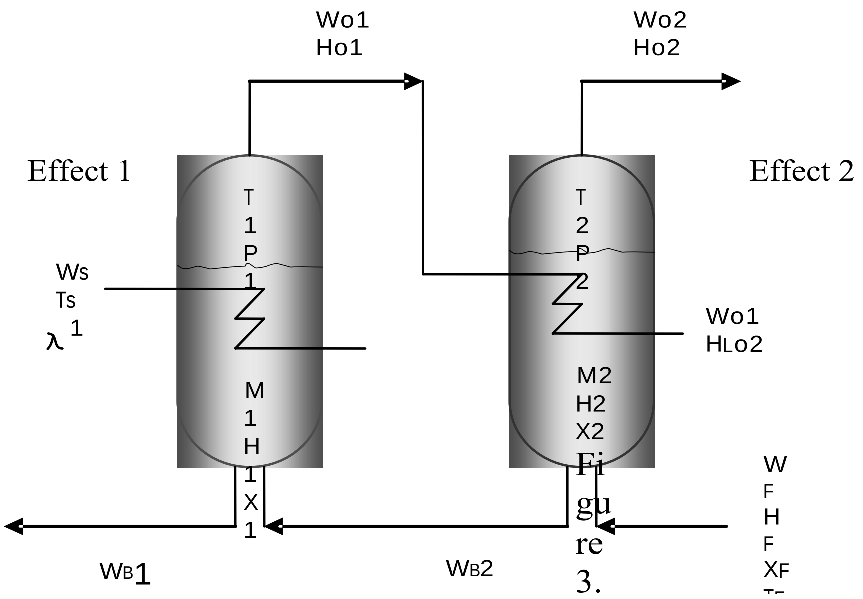

Fig. 2. Double effect evaporator with countercurrent flow.

Table 2. Procedures required for process definition for a model 'Mm'.

Procedure name	Description
`MmModel`	Evaluates constraints, balances and measurements given the inputs.
`MmCnfg`	Defines the pairing of measurements with manipulated variables
`MmInit`	Assigns conditions
`MmOper`	Defines operating strategy.
`MmExec`	Invokes the procedures to evaluate control criterion

.

The results are the diagnoses of control criteria (as presented in table 1) the scaling values and the coefficients for the precompensator

3. Cases of study

In order to illustrate the methodology described, we have applied it to a process of double effect evaporation, and to a distillation process.

3.1 Double effect evaporator

This process has wide range of applications in feeds and products, and allows the possibility of analyzing the interaction of streams: concurrent, counter current and the form of conveying the fluid from one vessel to another: either by pumping, or by its hydrostatic head. Here a double effect evaporator that has as main goal to purify trietilengycol is modeled. The effects are connected in counter current. The liquor is transported from second stage to the first one by hydrostatic pressure. Newell and Fisher (1972) presented a model of evaporator transporting the liquor by pumping. The enthalpy variation of steam is important in the energy balance of every evaporator, since natural transportation is more sensitive to disturbances of the saturation conditions. Low enthalpy steam is used as evaporation agent. Fig. 2 shows this process.

Table 3 lists the value of state variables and model parameters.

Table 3. Value of state variables and parameters for double effect evaporator.

	1^{st} Effect	2^{nd} Effect
Holdup,lb	98.36	138.625
Enthalpy,BTU/lb	170.733	164.13
Mass Fraction, TEG	0.0987	0.0484
Temperature, °F	703.1	692
Pressure, Psi	17.2	10.9
Bottom Flow(W_B), lb/min	2.25	4.625
Steam Flow (W_O),lb/min	2.38	2.375
Parameters:		
Transversal area, in^2	50.90	26.51
Heat transfer area, in^2	1,410	1,390
Kv constant	241,273	23,000
Total heat transfer coefficient (Kern, 1965)	1.74x10-2	$1.74x10^{-2}$

Table 4. Results of applying control criterion.

Number of candidates	Criteria
91 x 6	Total number of alternatives
13 x 9	Observability and controllability
47	Stability
2	Sensibility
2	Interaction between controls

Feed stream W_F = 7 lb/min, X_F= 0.032, H_F = 162 BTU/lb. At these operation conditions, we consider that only water is evaporated, and that steam is condensed in every heater. We also consider that steam holdup is negligible in every effect.

Inputs: $[W_F, H_F, X_F, W_S, \lambda_1]$
u = $[W_S, Hs, W_F, H_F]$
x = $[M_1, H_1, X_1, M_2, H_2, X_2]$
y = $[M_1, H_1$(from T_1), X_1, M_2, H_2(from T_2), X_2, W_F, H_F (from T_F) $P_1, P_2, W_{O1}, W_{O2}, W_{B1}, W_{B2}$]. From this sets we choose 2 for every configuration.

Equations

1^{st} Effect

$$dM_1/dt = W_{B2} - W_{B1} - W_{O1} \qquad \text{Mass}$$

$$d(M_1*X_1)/dt = W_{B2} *X_2 - W_{B1} *X_1 \qquad \text{Composition}$$

$$d(M_{11}*H_{11}+M_{1v}*H_{1v})/dt= W_{B2}*H_2-W_{B1}*H_1 - W_{O1}*H_{O1}+W_S*\lambda_1 \qquad \text{Energy}$$

2^{nd} Effect

$$dM_2/dt = W_F - W_{O2}- W_{B2} \qquad \text{Mass}$$

$$d(M_2*X_2)/dt = W_F*X_F - W_{B2} *X_2 \qquad \text{Composition}$$

$$d(M_{21}*H_{21}+M_{2v}*H_{2v})/dt= W_F*H_F-W_{O2}*H_{O2}- W_{B2}*H_2+W_{O1}*\lambda_2 \qquad \text{Energy}$$

Output flow equations are
For liquid $\Delta P_B = K_{vB} * W_B^2$
For steam $\Delta P_s = K_{vs} * W_s^2/\rho_s$

3.2. Correlations for thermodynamics and physical properties.

Water physical properties were obtained from quadratic correlations from Wark (1988). Enthalpy of mixture was approximated by a combination of the mass fraction without considering heat of mixing. From these polynomials, the derivatives $d\phi_s/dH_{ls}$ were also obtained. Total pressure was determined from the mass fraction contribution of vapor pressures.

$$P = X\,[P°_{TEG}(T)] + (1-X)\,[P°_{H2O}(T)]$$

From these measurements, we chose 2 variables to analyze control alternatives. Results in Table 4 we list the pairings of measurements and controls and how they were selected in the double effect evaporator.
The pairings that satisfied all control criteria were:

Pair 1: u = $[W_S, W_F]$, y = $[H_1, M_2]$
Pair 2: u = $[W_S, \lambda_{s1}]$, y = $[H_1, \lambda_{s1}]$

Given the type of measurements and the low process frequency (lower than 10/min), the first alternative was selected, since the ratio of singular values is closer to one. For this alternative the relation of singular values is $\sigma_{max}/\sigma_{min}$ = 12.55. The RGA for this configuration is: $\Lambda(0) = \begin{bmatrix} 0.58, 0.42 \\ 0.42, 0.58 \end{bmatrix}$

With this alternative, all the poles and zeroes are negative real. At $\omega > 5$ both diagonal elements of RGA are close to 1.
Process synthesis: The optimized weights for scaling according to eqn. 7 are:W = [0.9293 , 0.0707]. This reduces the objective function from 10.1 to 0.8. The first output is not diagonal dominant, and by contrast the second output is highly diagonal dominant. The optimized elements of the precompensator matrix according to eqn. 8 are:

$$K(s) = \frac{1}{s}\begin{bmatrix} -10.56\,s - 2.14 & 0.1\,s + 0.2111 \\ 4.767\,s - 0.4111 & -10.2\,s - 0.1 \end{bmatrix}$$

It produces a ten-fold increase in the diagonal dominance. Fig. 3 plots the column-diagonal dominance frequency for the coupled and the decoupled process in the range of frequencies of 0.005 – 5rad/min. The Gershgorin bands get narrower as frequency increases (Fig.4). The gains used according to the estimation of these bands for this model were:

$$F = \begin{bmatrix} 1/400 & 0 \\ 0 & 1/200 \end{bmatrix}$$

Fig. 5 presents the pole-zero map for the compensated model. Some zeroes lay close to the vertical axis, while the precompensator produced a small zero on the positive plane. Fig. 6 presents the dynamic behavior of this model under a step transient. The effect of decoupling has been achieved, but it requires a SISO regulator for stability.

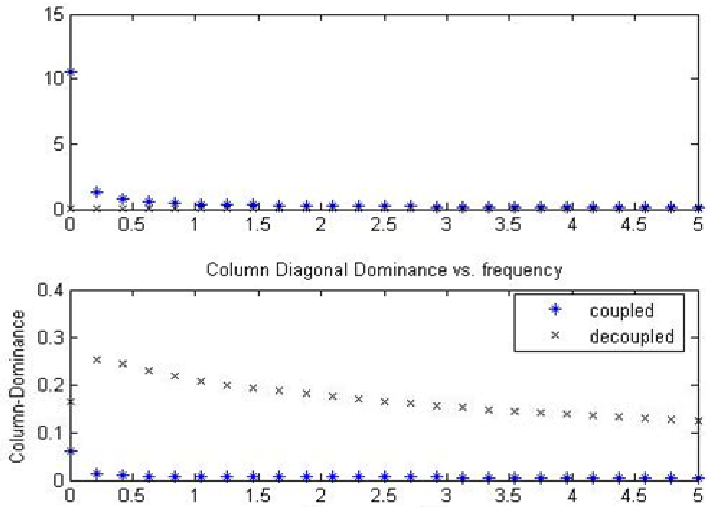

Fig.3. Column dominance vs. frequency.

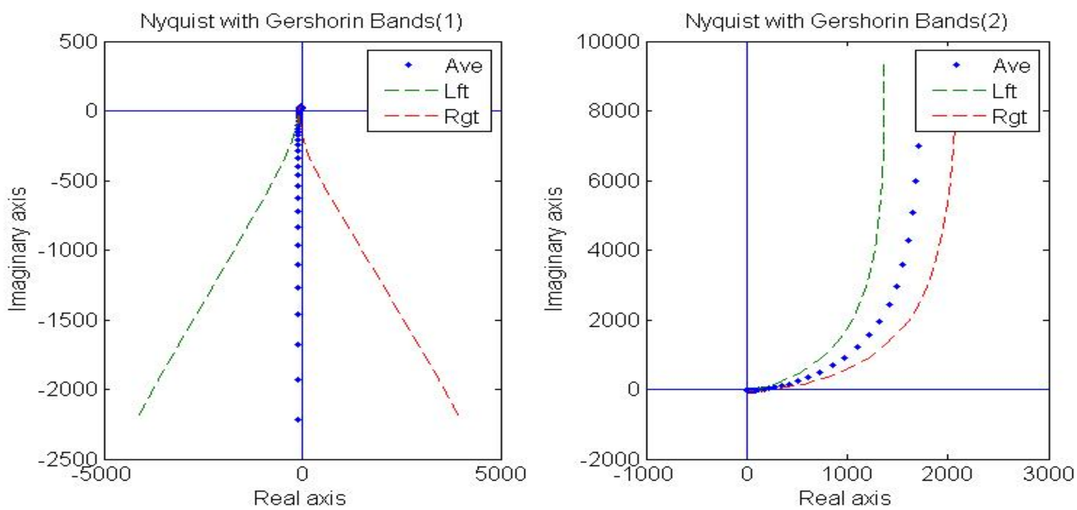

Fig. 4. Gershgorin bands for the double effect model.

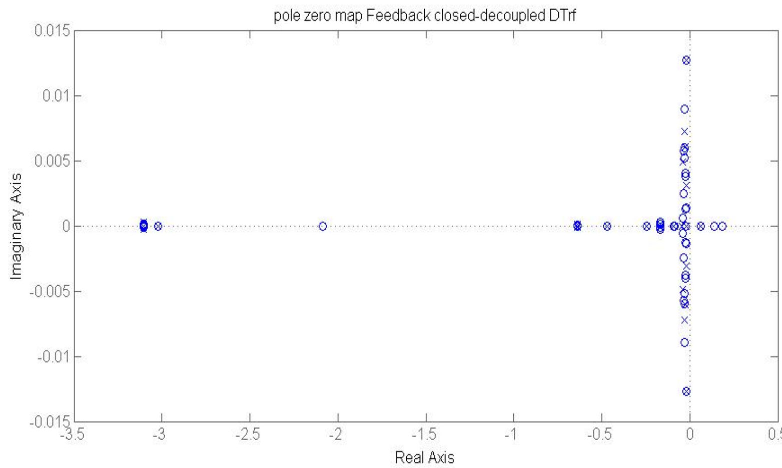

Fig. 5. Pole-zero map decoupled.

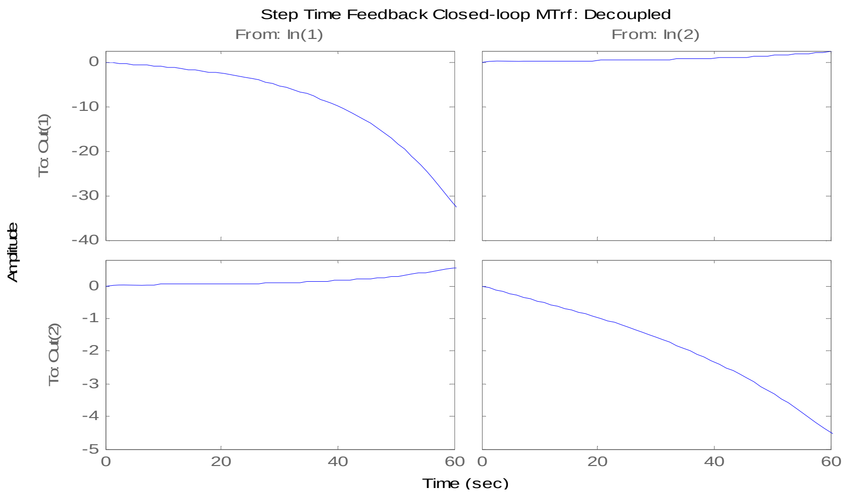

Fig. 6. Feedback-step transient.

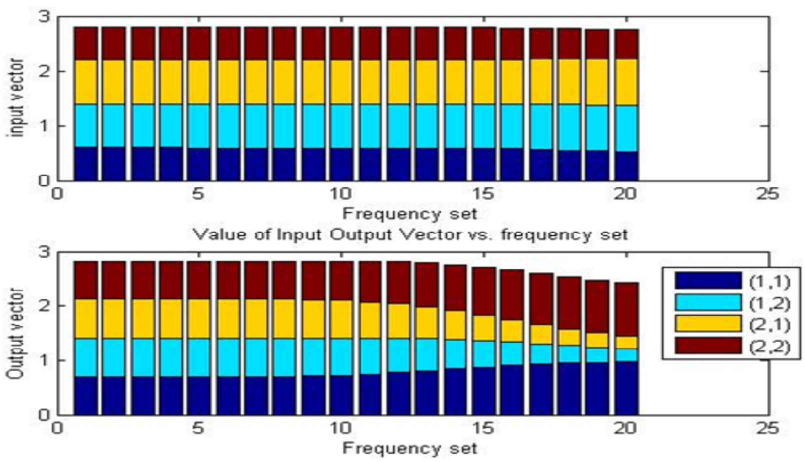

Fig. 7. Behavior of the input and output elements with frequency.

3.3. Distillation column

We use two types of distillation model

Column Model	Composition	Mass-composition, Energy
Composition balance	√	√
Mass balance		√
Energy balance		√
Hydraulics		√

3.3.1. Composition model

This model considers as state variables liquid composition only in every plate (Cingara and Jovanovic; 1990). The parameters for this model appear in Table 5.

Table 5. Operating conditions for composition model.

Number of Trays	11
Feed tray	6
α	2.00
x_F	0.450 mol
W_F	10.Kmol/min
x_B	0.104 mol
L/F	1.305
V/F	1.740

x = [x]
u = [L, V, B, D]
y = [X_B, Y_D]

Results: All poles and zeroes of the models are the left hand plane. The maximum ratio of singular values ($\sigma_{max}/\sigma_{min}$) = 22, occurs at low frequencies. Fig. 7 presents a stacked bar diagram of the elements of the input and output values of the elements of the SVD ($U\Sigma V^T$) in a range of frequencies of 0.005-14 rad/s. The output vector is more sensitive to frequency changes than the specified input vector. Thus, stability does not present a difficulty for this process-model; scaling is in the range of sensitivity level of current instruments. However, the interaction between the manipulated variables presents a challenge, since from Fig. 8 we observe that the strength of interaction changes with frequency. Both diagonal elements have similar magnitude. When decoupling is used, the following results are obtained.

Scaling vector $W = [0.5154 \quad 0.4846]$, the preconditioning matrix for a PI compensator is:

$$K(s) = \frac{1}{s}\begin{bmatrix} -0.0556\ s - 0.8835 & -1.967\ s - 0.1 \\ -0.8111\ s - 0.7395 & -2.500\ s - 0.1 \end{bmatrix}$$

Fig. 9 shows the result of Column Diagonal Dominance vs. frequency. We can appreciate that dominance is achieved.

Fig. 10 Present the bands of the direct Nyquist Array. This indicates that ($-1/f_{11} <$ 1.e-03; $1/f_{22} <$ 1e-04).

Fig. 11 shows the dynamic results for a step for the uncoupled process with $F = \begin{bmatrix} 1. & 0 \\ 0 & 1. \end{bmatrix}$.

The interactions in the linearized model have effectively been reduced. Additional SISO controls are required to maintain stability.

3.3.2. Mass-energy-species model

This model represents a distillation column to separate an azeotropic mixture of Ethanol-Water. We use as a reference the model described by Gani et al. (1986). The following considerations were made for constructing this model (Fig. 12):

Phase equilibrium was evaluated by Peng-Robinson-Stryjek-Vera Eq. of state (1986).

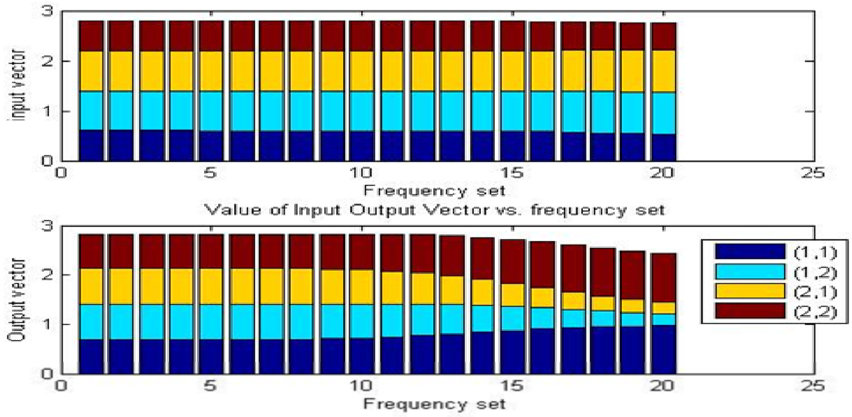

Fig. 7. Behavior of the input and output elements with frequency.

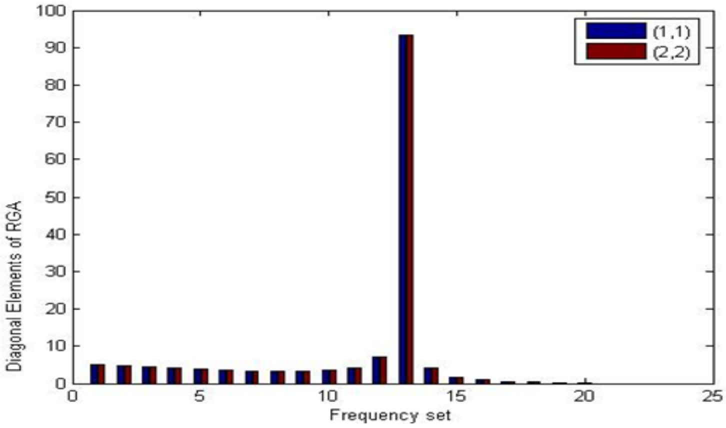

Fig. 8. Diagonal elements of RGA vs. frequency.

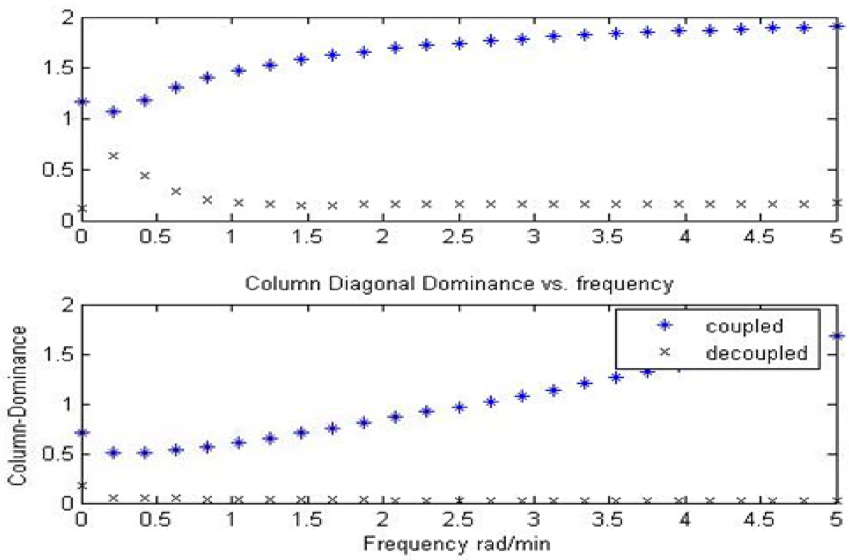

Fig. 9. Column dominance vs. frequency.

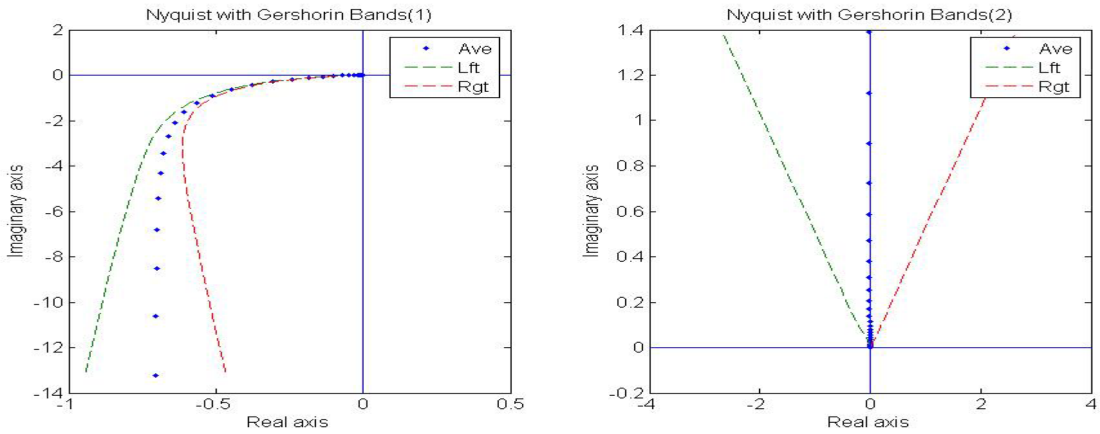

Fig. 10. Gershgorin bands.

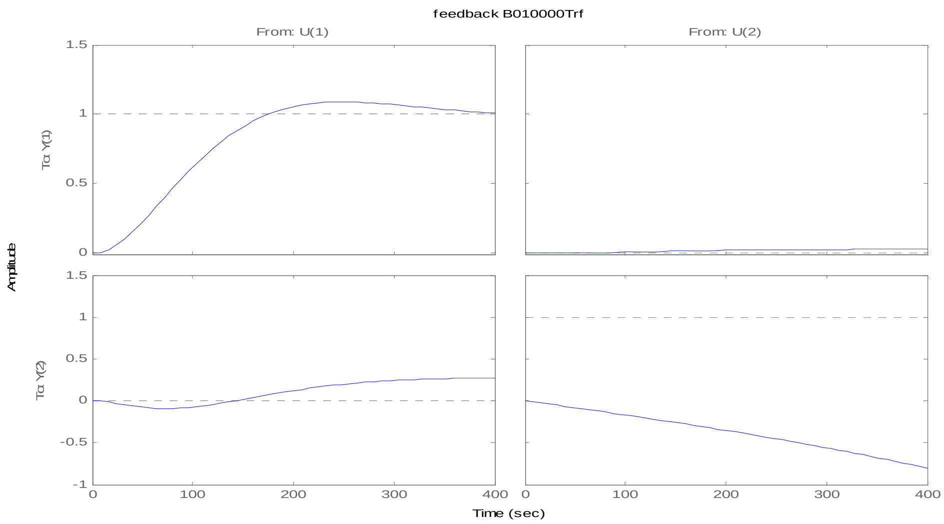

Fig. 11. Dynamic result of feedback step.

PRSV-EOS was also used to evaluate liquid and vapor density and enthalpy. The interaction parameters were fitted from experimental data (Barragan, 1995). Murphee efficiency was assumed. Vapor holdup is neglected. Reverse vapor flow is allowed. Hydraulics follows a modification of Francis weir equation. Hydraulics considers a two-phase in the holdup tray (Benett, Agrawal, Cook, 1983). Reboiler and condenser heat transfer dynamics were neglected. Exit pressure of products is fixed. Tray dynamics are included in the model to represent flow oscillations and dangerous conditions like tray drying. The effect of energy is important, especially in components with different heat of vaporization. This model presents a delayed –coupling. For instance, closing the condenser valve would eventually pressurize the column; as a result, bottom product would increase. As noted by Luyben (1970), when both products are important, to reduce the interaction, the geometry of the process (more trays, large feed tanks), or its operation (large reflux ratios) can be modified which would increase capital or operating costs. The parameters for this model appear in Table 6.

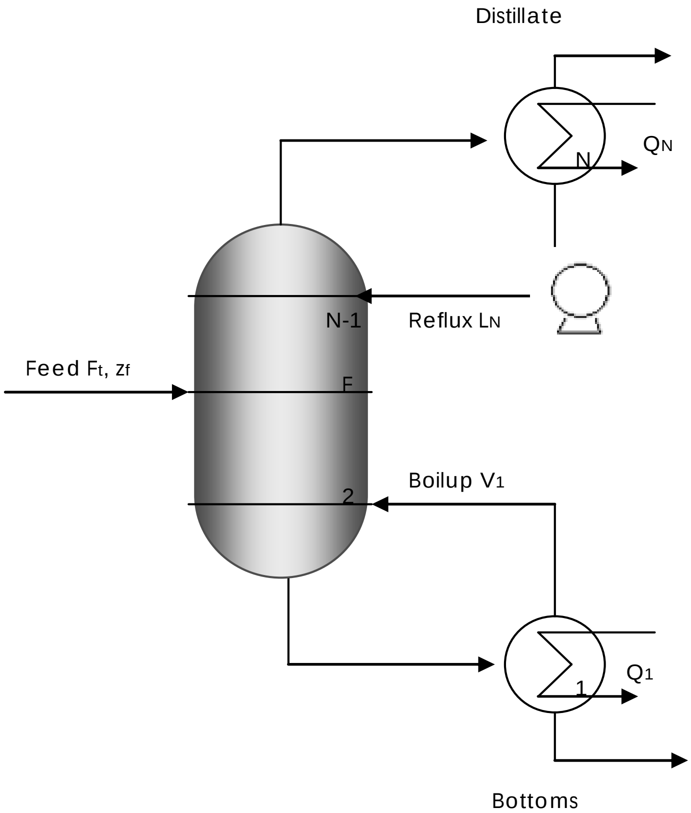

Fig. 12. Distillation column.

Table 6. Operating conditions for distillation column.

Type of condenser	Total
Tray type	Sieve
Number of Trays	14
x_F	0.30018 mol EthOH/mol tot
W_F	5 Kgmol/min
Feed tray	4
Feed condition	Saturated liquid at feed tray
Boiler Area	15 m^2
Boiler Pressure	101.325 KPa
Q_B	2334,30KJ/min
Weir height	0.0381 m
Condenser Area	5 m^2
Condenser Pressure	60.7950 KPa
Q_D	2297,30.KJ/min
L/F	0.9901
V/F	1.1883

x: [M_T,U_T, X_T] of ethanol in every plate.
Inputs:[F,Z_F, D, B ,Q_B, Q_D]
y:[X_B, X_D]
u:[D, B, L, V, Q_B, Q_D]

Of the 4 manipulated variables 2 variables must be used to control the level in the reboiler and in the condenser, 2 manipulated variables remain, from these we selected [Q_B, L,] as manipulated variables. Solution of every stage was numerically solved every plate to compute pressure, temperature and all the temporal derivatives. Liu (1983) noted that distillation presents a singularity at the steady state, since, from the global mass balance we obtain.

$$\begin{bmatrix}\Delta x_B \\ \Delta y_D\end{bmatrix} = (y_d - x_b)^2 \begin{bmatrix} -\dfrac{1}{(z-y_d)} & +\dfrac{1}{(z-y_d)} \\ +\dfrac{1}{(z-x_b)} & -\dfrac{1}{(z-x_b)} \end{bmatrix} \begin{bmatrix}\Delta B \\ \Delta D\end{bmatrix} \quad (9)$$

Near the steady state, this model presents difficulty in the numerical solution if the solution was solved sequentially, i.e. first solve constitutive equations eqn. 1, and second solve the balance equations eqn 3. Since the solution requires an inner –outer iteration cycles, both iteration cycles require numerical derivative. For high accuracy when both types of equations were solved simultaneously in semi-explicit form with the code ode23t of Matlab, the simultaneous solution was one order of magnitude faster than the sequential solution.

Conditioning number grows with frequency; at $w = 200$ its value is 10,000. In Fig. 13 we appreciate that while the input vector changed with frequency, the output vector is almost constant, with large off-diagonal elements. We observed that the strength of diagonal dominance depends upon the selected ranges of frequencies. At low frequencies it is not possible to obtain diagonal dominance for this model. At small frequencies not even, a PID compensator is able to produce diagonal dominance. The selected range of frequencies is [1, 2, 3, 5, 10]. The scaling vector obtained is W = [0.0774, 0.9226]. The optimized PI compensator obtained by eqn 8 (with a value of objective function equal to 0.53) is:

$$K(s) = \frac{1}{s}\begin{bmatrix} 12.78\,s + 0.6144 & -1.294\,s + 1.995 \\ 0.0652\,s + 0.0306 & 0.0884\,s - 0.05775 \end{bmatrix}$$

Fig. 14 present the Nyquist diagrams, from them we selected as gains:

$$F = \begin{bmatrix} 1/200 & 0 \\ 0 & 1/10 \end{bmatrix}$$

Fig. 15 presents the transient from t = 0-10 min for a step response. After a time delay, we obtain an oscillatory-sharp response.

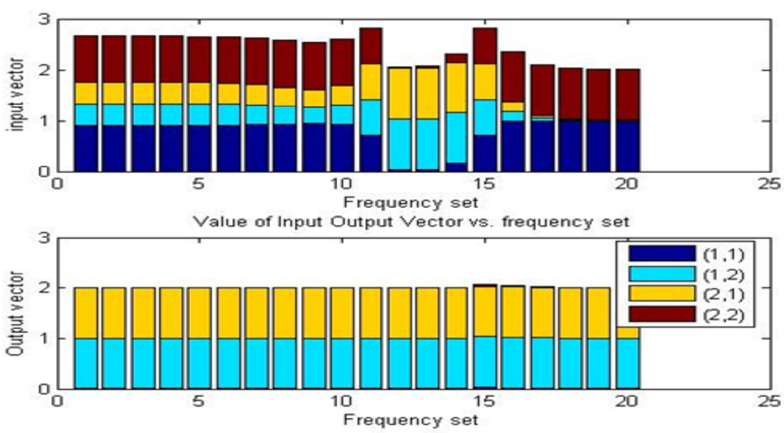

Fig. 13. Input, output elements vs. frequency.

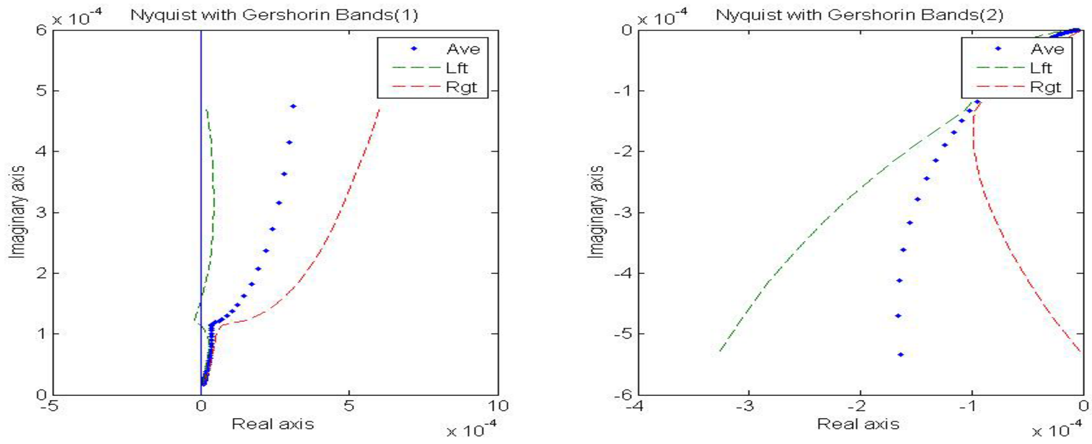

Fig. 14. Gershgorin bands

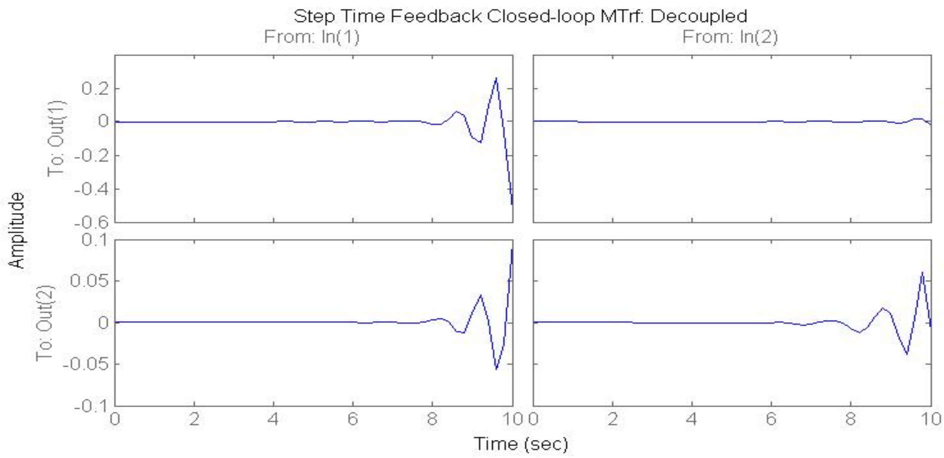

Fig. 15. Feedback step response

Conclusions

Pairing of input output based on singular values is adequate for systems that do not vary strongly with frequency. Design patterns automatize evaluation of control criteria, such as observability, stability, and scaling. It also reduces potential coding error, together with the available linearization functions in Matlab. Decoupling depends upon the frequency range. The net effect of decoupling is an amplitude increase. In the first and the last example, the stability is deteriorated with a decoupling precompensator. The methodology described in this work easies the analysis of control alternatives. This methodology allows focusing on model development and validation at steady and dynamic state, since many control alternatives can be evaluated automatically. The supreme goal is that given a process model and its control objectives, the best alternatives can be generated from the analysis of the computer model. Diagonal dominance allows analyzing which inputs have larger influence on a given output, and scales the control by a diagonal compensators. In addition, this compensator easies the change of automatic to manual operations, which is one of the warnings of automatic control given by Rosenbrock (1997).

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Nomenclature

A: state-state matrix
B: state-inputs matrix. Bottom flow
C: output-state matrix
D: output-input matrix. Distillate flow
E: Internal Energy
e: output error
F: feed flow
f: time dependent function
G: Process transfer matrix
g: measurements function
H: Enthalpy
K: Compensator matrix
L: Liquid flow
M: Mass holdup
M: Matrix
n: number of states
P: Pressure
p: parameter vector
Q: = G*K
r: constraints
t: time
T: Temperature
U: Heat transfer coefficient
u: input vector
V: Vapor flow
W: Mass flow
w: scaling vector
X: mol fraction
x: state vector
y: output vector

Greek letters:

α: relative volatility
σ: singular value
Σ: singular matrix
ϕ: Physical property.
λ: Water latent heat at pressure and temperature of heating steam

Subscripts

1: First effect
2: Second effect
B: Bottom
Boi: Boiler
Fed: Fed
F: Feed
L: liquid
l: lower
Max: maximum
Min: Minimum
O: Output steam
P: Plate
R: reduced
Rec: Recuperation section
S: Input steam
Stp: Spliting section
Tot: Total
U: upper
V: Vapor

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